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Chiral pions in a magnetic background

G. Colucci^{a,*}, E.S. Fraga^{a,b,c}, A. Sedrakian^a^a Institute for Theoretical Physics, J.W. Goethe-University, D-60438 Frankfurt am Main, Germany^b Frankfurt Institute for Advanced Studies, D-60438 Frankfurt am Main, Germany^c Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 21945-970, Brazil

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ABSTRACT

We investigate the modification of the pion self-energy at finite temperature due to its interaction with a low-density, isospin-symmetric nuclear medium embedded in a constant magnetic background. To one loop, for fixed temperature and density, we find that the pion effective mass increases with the magnetic field. For the π^- , interestingly, this happens solely due to the trivial Landau quantization shift $\sim |eB|$, since the real part of the self-energy is negative in this case. In a scenario in which other charged particle species are present and undergo an analogous trivial shift, the relevant behavior of the effective mass might be determined essentially by the real part of the self-energy. In this case, we find that the pion mass decreases by $\sim 10\%$ for a magnetic field $|eB| \sim m_\pi^2$, which favors pion condensation at high density and low temperatures.

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1. Introduction

The behavior of hadronic matter in a medium under the influence of a strong external magnetic field can be very rich and subtle, and has been the subject of intense investigation in the last few years. In fact, in-medium strong interactions under extreme magnetic fields are of experimental relevance in heavy ion collisions and in astrophysics, exhibit a rich new phenomenology and are amenable to lattice simulations. (For comprehensive reviews, see Ref. [1].)

Even if every model calculation has predicted that large enough magnetic fields, typically of the order of a few times m_π^2 , could bring remarkable new features to the thermodynamics of strong interactions, from shifting the chiral and the deconfinement crossover lines in the phase diagram [2–15] to transforming the vacuum into a superconducting medium via ρ -meson condensation [16,17], essentially all models fail to describe coherently the available lattice data [18–21]. The reasons for that are still unclear, although there are some indications that confinement plays a relevant role [15,22], which is not captured in the usual low-energy effective chiral models of QCD [23]. In any case, the situation calls for theoretical investigations in more controlled setups, with less

freedom and parameters to adjust. This approach has proved to be fruitful in the large- N_c [22] and perturbative [24] limits of QCD: in the former, the behavior of the critical temperature for deconfinement was found to be in qualitative agreement with lattice data; in the latter, a trivial chiral limit for the two-loop contribution to the QCD pressure in a strong magnetic background was revealed.

Following this line of action, a natural extension is the study of hadronic matter in the complementary, low-energy sector, in the presence of a strong magnetic field, in a controlled setup. Thus, since we are interested in the low-density, low-temperature sector of the phase diagram of nuclear matter, we adopt the framework of chiral perturbation theory, which represents a powerful tool to study the low-energy regime of the pion-nucleon physics [25].

It is the purpose of this work to investigate some properties of isospin-symmetric nuclear matter in the limit of low density and temperature, embedded in a strong magnetic background. In particular, we study the modifications of the spectrum of the lowest energy degree of freedom, the pion, due to the interaction with nucleons and the constant magnetic field. More specifically, we compute the pion effective mass in the presence of a constant magnetic field to one loop. (Even if we do not address the phase diagram here, it should be mentioned that the inclusion of nucleons, and pion-nucleon interactions, proved to be necessary for a satisfactory description of the behavior of the deconfinement critical temperature as a function of the pion mass and isospin [26].) For this purpose, we consider fully relativistic chiral perturbation theory as a framework for our computation. This is needed to define consistently the fermion propagators in a magnetic background. At the same time, this work extends a previous treatment

* Corresponding author.

E-mail addresses: colucci@th.physik.uni-frankfurt.de (G. Colucci), fraga@if.ufrrj.br (E.S. Fraga), sedrakian@th.physik.uni-frankfurt.de (A. Sedrakian).

whereas the neutron loop contribution is not affected by the presence of the magnetic field

$$\Pi_n^{\text{WT}}(Q) = \frac{1}{f_\pi^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\omega E_p - \mathbf{p} \cdot \mathbf{q}}{E_p} n_F(E_p - \mu). \quad (9)$$

In Eq. (8) $n_F(x) = (e^{x/T} + 1)^{-1}$ is the Fermi distribution, $E_3 = \sqrt{p_3^2 + m^2}$, m being the proton mass, and the subscript L indicates that the vectors live in the two-dimensional subspace defined by the time component and the space component that is aligned with the magnetic field, i.e. $\mathbf{p}_L = (p_0, p_3)$.

The WT self-energy for the positively charged pion will be just the opposite of Eq. (7). Finally, the π^0 does not receive any one-loop contribution from the WT interaction term.

The contribution to the pion self-energy from the one-pion exchange term in the Lagrangian (2) is, on the other hand, quite involved. For the charged pion one has that the two nucleons in the diagram correspond to two different isospin states, one being a proton and the other a neutron. For the proton we choose now a more convenient form for the propagator [38], which in the LLL approximation reads

$$\begin{aligned} S_{\text{LLL}}^{(p)}(X, Y) &= \int \frac{dp_0 dp_2 dp_3}{(2\pi)^3} \\ &\times \sqrt{\frac{|eB|}{\pi}} e^{-ip_0(x_0 - y_0) + ip_2(x_2 - y_2) + ip_3(x_3 - y_3)} \\ &\times \exp\left\{-\frac{|eB|}{2} \left[\left(x_1 - \frac{p_2}{eB}\right)^2 + \left(y_1 - \frac{p_2}{eB}\right)^2 \right]\right\} \\ &\times \mathcal{P}_0 \frac{\mathbf{p}_L \cdot \boldsymbol{\gamma}_L + m}{\mathbf{p}_L^2 - m^2}, \end{aligned} \quad (10)$$

where $\mathcal{P}_0 = \frac{1}{2}[1 - i\gamma^1\gamma^2 \text{sign}(eB)]$. The self-energy in the coordinate space is given by

$$\begin{aligned} \Pi^{\text{OPE}}(X, Y) &= -\frac{g_A^2}{f_\pi^2} \text{Tr}[\gamma_5 \gamma^\mu S_p(X, Y) \gamma_5 \gamma^\nu S_n(Y, X) Q_\mu Q_\nu], \end{aligned} \quad (11)$$

where Q_μ and Q'_ν are defined in the momentum space as the pion momenta

$$\Pi^{\text{OPE}}(Q, Q') = \int d^4X d^4Y e^{i(Q' \cdot X + Q \cdot Y)} \Pi^{\text{OPE}}(X, Y). \quad (12)$$

By substituting the propagator in Eq. (10), one can write

$$\Pi^{\text{OPE}}(Q, Q') = (2\pi^3) \delta^{(0,2,3)}(q' + q) \tilde{\Pi}^{\text{OPE}}(Q), \quad (13)$$

where

$$\begin{aligned} \tilde{\Pi}^{\text{OPE}}(Q) &= \frac{g_A^2}{f_\pi^2} \sqrt{\frac{16\pi}{|eB|}} Q_\mu Q_\nu \int \frac{d^4P}{(2\pi)^4} \\ &\times \frac{F^{\mu\nu}(P)}{(P^2 - m^2)[(\mathbf{p} + \mathbf{q})_L^2 - m^2]} \\ &\times \exp\left\{-\frac{1}{eB}[(q_1 + p_1)^2]\right\} \end{aligned} \quad (14)$$

and

$$\begin{aligned} F^{\mu\nu}(P) &= (\mathbf{p} + \mathbf{q})_L^\mu P^\nu + P^\mu (\mathbf{p} + \mathbf{q})_L^\nu \\ &- g^{\mu\nu}[(\mathbf{p} + \mathbf{q})_L \cdot \mathbf{p}_L + m^2]. \end{aligned} \quad (15)$$

Notice that, in Eq. (14), we used the fact that the δ -function and the on-shell condition for the pion lead to $q_1 + q'_1 = 0$.

Since the pole structure of the propagators was not modified, the self-energy at finite temperature is obtained by replacing the time component of the four momenta by the appropriate (fermionic or bosonic) Matsubara frequency, i.e. $P^\mu = (p_0, \mathbf{p})$, with $p^0 = ip_n + \mu$, $ip_n = 2n\pi iT$ for the pion and $ip_n = (2n + 1)\pi iT$ for the nucleon, $n \in \mathbb{Z}$ and T being the temperature.

Performing the sum over the fermionic Matsubara frequency, the retarded self-energy reads

$$\begin{aligned} \tilde{\Pi}^{\text{OPE}}(Q) &= -\frac{g_A^2}{f_\pi^2} \sqrt{\frac{16\pi}{|eB|}} Q_\mu Q_\nu \int_{-\infty}^{\infty} \frac{d^3\mathbf{p}}{(2\pi)^3} e^{-\frac{1}{eB}(q_1 + p_1)^2} \\ &\times \frac{1}{4E_p E_{pq}^L} [F^{\mu\nu}(E_p, \mathbf{p}) n_F(E_p - \mu) \\ &- F^{\mu\nu}(E_{pq}^L - q_0, \mathbf{p}) n_F(E_{pq}^L - \mu)] \\ &\times \frac{1}{q_0 + E_p - E_{pq}^L + i\eta}, \end{aligned} \quad (16)$$

where $E_{pq}^L = \sqrt{(\mathbf{p}_L - \mathbf{q}_L)^2 + m^2}$. Notice that in this case, due to the presence of a neutral field in the loop, one does not have the dimensional reduction which takes place in the case of the gluon self-energy in QCD [39–41]. This is instead the case for the π^0 , as we shall see in the following.

We separate the imaginary and real parts of the self-energy in Eq. (16) by means of the Sokhotski–Plemelj formula. Since the imaginary part of the self-energy satisfies the hypothesis of the Kramers–Kronig dispersion relation, the real part can be computed as

$$\text{Re } \Pi^{\text{OPE}}(\omega, \mathbf{q}) = \frac{1}{\pi} \oint_{-\infty}^{\infty} d\omega' \frac{\text{Im } \Pi^{\text{OPE}}(\omega', \mathbf{q})}{\omega' - \omega}. \quad (17)$$

As before, the result for the positively charged pion will be the opposite as compared to the π^- .

The one-loop contribution to the neutral pion self-energy is the sum of the proton and the neutron loops. The neutron loop is not affected by the presence of the magnetic field and vanishes in the limit of $\mathbf{q} \rightarrow 0$. The proton loop can be computed in a way that is very similar to the charged pion self-energy computation. The result reads

$$\begin{aligned} \Pi_0(Q, Q') &= -\frac{g_A^2}{2f_\pi^2} (2\pi)^4 \delta^4(Q' + Q) \frac{|eB|}{2\pi} e^{-\frac{q_1^2 + q_2^2}{2|eB|}} Q_\mu Q_\nu \\ &\times \int \frac{dp_3}{2\pi} \frac{1}{4E_p^L E_{pq}^L} [H^{\mu\nu}(E_p^L, p_3) n_F(E_p^L - \mu) \\ &- H^{\mu\nu}(E_{pq}^L - iq_n, p_3) n_F(E_{pq}^L - \mu)] \\ &\times \frac{1}{E_p^L - E_{pq}^L + iq_n}, \end{aligned} \quad (18)$$

where

$$\begin{aligned} H^{\mu\nu}(\mathbf{p}_L) &= \mathbf{p}_L^\mu (\mathbf{p}_L + \mathbf{q}_L)^\nu + \mathbf{p}_L^\nu (\mathbf{p}_L + \mathbf{q}_L)^\mu \\ &- g^{\mu\nu}[\mathbf{p}_L \cdot (\mathbf{p}_L + \mathbf{q}_L) + m^2]. \end{aligned} \quad (19)$$

As already stated previously, in this case we recover the dimensional reduction, from $(3 + 1)$ to $(1 + 1)$, where the only spatial dimension that is relevant for the dynamics is determined by the direction of the magnetic field.

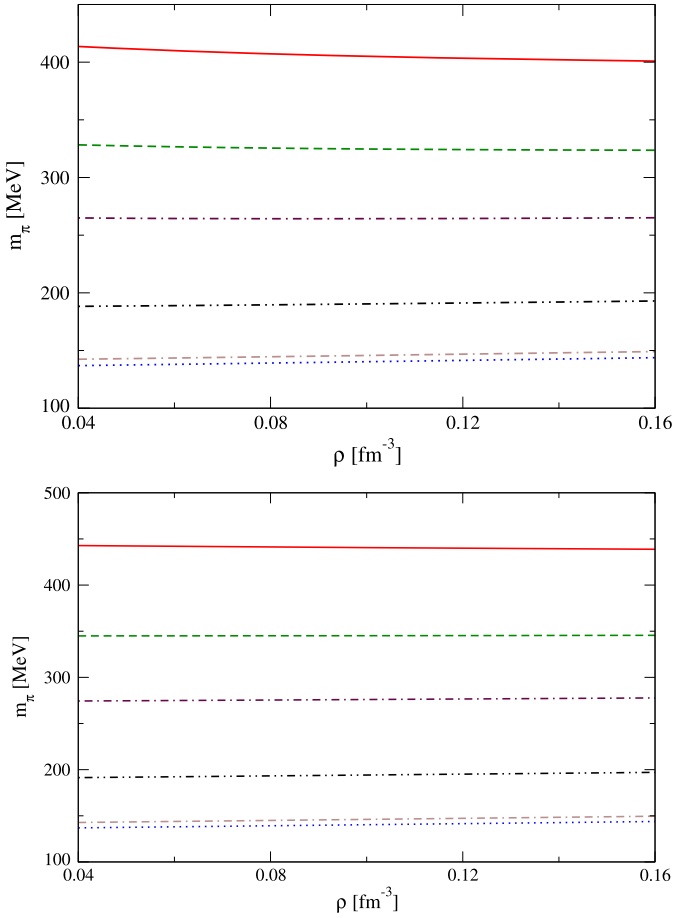


Fig. 3. (Color online.) Upper panel: π^- effective mass in a constant magnetic background, as a function of the density, for zero temperature. From below, lines correspond to increasing values of the magnetic field, namely $eB/m_\pi^2 = 0.01, 0.1, 1, 3.2, 5.6, 10$. Lower panel: same as above, for $T = 50$ MeV.

4. Results

We compute numerically the integrals appearing in Eqs. (7), (16) and (18). In the limit of $\mathbf{q} \rightarrow 0$ the only nonvanishing contribution to the self-energy is given by the WT contribution in Eq. (7). This is due to the fact that in symmetric nuclear matter the poles in Eq. (16) and (18) vanish for $\mathbf{q} \rightarrow 0$, leading thus to a vanishing self-energy. So, the neutral pion, whose self-energy comes only from the one-pion exchange term in the Lagrangian, has its mass unaltered as is also the case when no magnetic background is present.

Fig. 3 shows the solution of Eq. (6) in the LLL approximation for the negatively charged pion, in the case of isospin-symmetric nuclear matter, as a function of the density. The upper panel corresponds to the case of zero temperature. Clearly, the main contribution to the effective mass is given by the term proportional to $|eB|$ in Eq. (6) that represents the trivial Landau quantization shift. The variation with the density is almost negligible. Nevertheless, one can notice that in the case of low magnetic fields the effective mass slightly increases with the density, while in the case of extremely strong magnetic fields the mass decreases. Therefore, if we would plot only the self-energy contribution to the effective mass, it would exhibit an appreciable drop for high magnetic fields. We should, of course, stress that the extremal values of the magnetic field magnitude shown in Fig. 3 might bring some inconsistency due to the fact that for too low fields one can not apply the LLL approximation, whereas for extremely strong fields one has

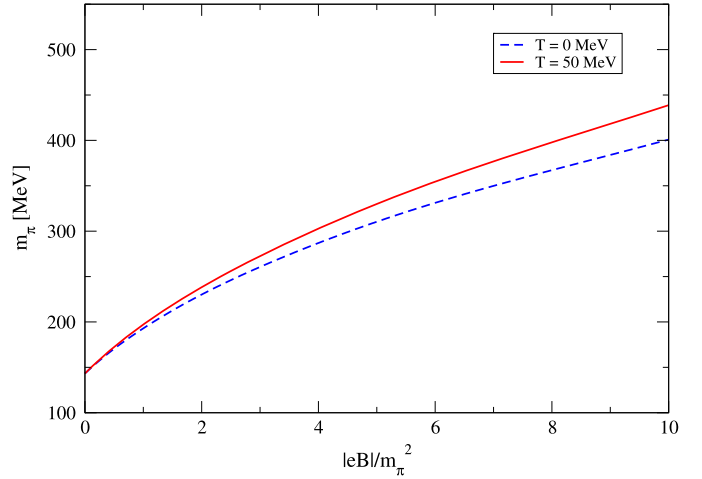


Fig. 4. (Color online.) π^- effective mass as a function of the magnetic field at saturation density, $\rho = 0.16 \text{ fm}^{-3}$. The (blue) dashed line is the result for the zero temperature case, while the full (red) line corresponds to a temperature $T = 50$ MeV.

to treat more carefully the chiral power counting. Yet, we believe that Fig. 3 illustrates clearly the qualitative trend as one comes from low to high magnetic fields.

In the lower panel the effect of the temperature is included. The main features of the plot remain the same but, at equivalent values of the magnetic field, one can see that these curves lie above the corresponding curves at zero temperature. This is also shown in Fig. 4, in which we fix the density at nuclear saturation, $\rho = 0.16 \text{ fm}^{-3}$, and vary the magnetic field. The dashed curves correspond to the zero temperature case, while the full lines to $T = 50$ MeV, and one can see that, for fixed density and temperature, the pion mass increases steeply with the external magnetic field.

As remarked previously, the result for very strong magnetic fields ($|eB| \geq m_\pi^2$) has to be considered as an extrapolation. Indeed, for such high values of the magnetic field, the chiral power counting does not hold anymore. Since the low-energy scale of the theory (the pion mass) and the hard scale become comparable, one should take into account higher order diagrams that become, in principle, relevant in this case. Nonetheless, the trend seems clear from within the region of validity of our approach and connects smoothly to the regions of higher and lower fields, which is encouraging.

To unveil the role played by the real part of the self-energy contribution to the mass of the π^- , we compute its effective mass having subtracted the trivial shift due to the presence of the magnetic background (Landau quantization), namely we solve

$$\bar{m}_\pi^2 = m_\pi^2 - \text{Re} \Pi(\bar{m}_\pi^2, \mathbf{q} = 0; \mathbf{B}). \quad (20)$$

Fig. 5 displays our results for \bar{m}_π as a function of the magnetic field, and shows a significant decrease in the effective mass of the π^- as a function of the magnetic field. This effect is, of course, diminished as the temperature is increased.

The phenomenological motivation comes from physical systems with different charged particle species in the presence of moderately strong magnetic fields, as can be found *e.g.* in compact stars. In this case, one has to take into account the contributions coming from higher Landau levels, at least the first nontrivial ones. Thus, the trivial shift in the spectrum of different (charged) fermions and bosons will be of the same order ($\sim |eB|$), and the contribution that will be relevant for the behavior of the effective mass will possibly be determined essentially by the real part of the self-energy. In this context, we find that for the negatively charged pion the effective mass is lowered by the presence of a strong magnetic

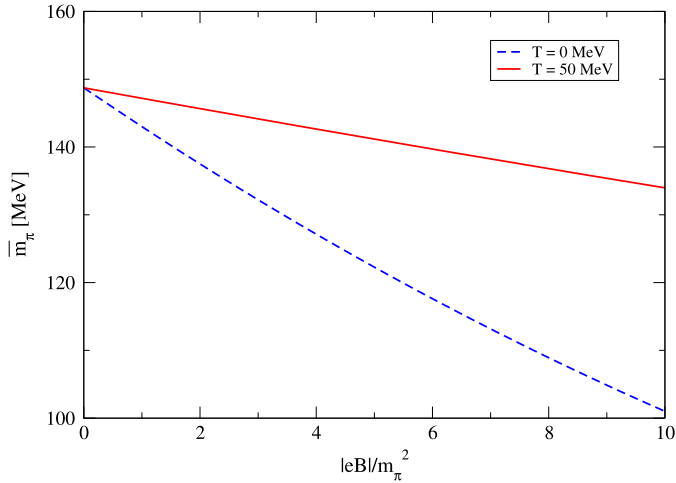


Fig. 5. (Color online.) Solution of Eq. (20) as a function of the magnetic field, at saturation density $\rho = 0.16 \text{ fm}^{-3}$. The (blue) dashed line is the result for the zero temperature case, while the full (red) line refers to a temperature $T = 50 \text{ MeV}$.

background. Due to this feature, properties of a system involving dense nuclear matter and leptons, as can be found in compact stars and supernovae [42,33], might change significantly, depending on the behavior of the spectrum of charged fermions.

5. Summary

We have investigated hadronic matter in the low-energy sector in the presence of a strong magnetic field in the controlled setup of chiral perturbation theory. More specifically we have studied the modification of the pion self-energy at finite temperature due to its interaction with a low-density, isospin-symmetric nuclear medium in the presence of an external magnetic field.

To one loop, for fixed temperature and density, we found that the pion effective mass increases steeply with the magnetic field, a result that is enhanced when temperature is included, as expected. As a function of the density, on the other hand, the behavior of the effective mass is quite flat for different values of the field. However, even keeping in mind the caveat of our method when considering too low or too high fields, it seems clear that there is a qualitative change in the overall behavior: augmentation of the effective mass for low fields and depletion for high fields. The latter effect is, of course, hampered as we increase the temperature.

A subtle point that can play a relevant role in some actual physical systems, with different charged particle species in the presence of moderately strong magnetic fields, is the fact that the increase in the effective mass of the π^- with the magnetic field is due solely to the trivial Landau quantization shift $\sim |eB|$, since the real part of the self-energy is negative in this case. As was shown in the previous section, if we subtract the former trivial effect, the effective mass of the negatively charged pion drops considerably with the magnetic field. If all charged particles undergo an approximately equivalent trivial shift of this sort, the modifications that may be relevant, phenomenologically, are those brought about by the real part of the respective self-energies. In this case, we find that *the pion mass decreases by $\sim 10\%$ for a magnetic field $|eB| \sim m_\pi^2$, which favors pion condensation at high density and low temperatures.* Such scenario may take place in neutron star matter and supernovae and requires further investigation.

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